Estimation of signals in white noise using neural networks modeling

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Abstract

This work examines the use of artificial intelligence in Signal Processing, its application and results. Artificial intelligence is used to reconstruct noisy signals. Due to the use of artificial intelligence, the time and efficiency of signal processing has been increased. Methodology: To solve this problem, we modeled the noisy signal beforehand. First, this signal was reconstructed based on traditional methods, that is, cleaned of noise. A neural network was trained based on the noisy signal and the signal reconstructed using conventional methods. In this case, incoming database noise signals were used for the network, and regenerated signals were used for the outgoing database. "Adam"optimizer and "Mean squared error"error function were used for the neural network.

Introduction

In this paper, we consider the problem of statistical signal processing using neural networks. Let the observed process obeys following stochastic differential equation

$$dy_t = s_t dt + \varepsilon dw_t, 0 \leq t \leq 1, \qquad (1)$$

where S_t is an unknown signal, (y_t) is observations, $\varepsilon > 0$ is intensity noise, the noise process (w_t) is Brownian motion (Wiener process).

Traditional Statistical method

To estimate the function S in equation 1, we use the following $(\hat{S}^*_\lambda)_{\lambda\in\Lambda}$ family of improved weighted least squares estimators, that is

$$\hat{S}_\lambda(x) = \sum_{j=1}^{n} \lambda_j \hat{\theta}_j \phi_j(t), \tag{2}$$

where λ is vector of weight coefficients $\lambda = (\lambda_1, ..., \lambda_n)$ belongs to some finite set $\Lambda \subset [0,1]^n$ with $n \geq 3$,

Traditional Statistical method

θ- Fourier coefficients are as follows

$$\theta_j = \frac{1}{T}\int_0^T \phi_j(t)\, dy_z = \frac{1}{n}\sum_{k=1}^{n}\phi_j(t_k)\Delta y_{t_k} \tag{3}$$

and ϕ_j is a basis in $\mathcal{L}_2(0,1)$ space and it is $\phi_1 = 1$, and for $j \geq 2$

$$\phi_j(t) = \begin{cases} \sqrt{2}sin(2\pi[\frac{j}{2}]t), & \text{for even j} \\ \sqrt{2}cos(2\pi[\frac{j}{2}]t), & \text{for odd j} \end{cases} \tag{4}$$

where $[a]$ is the integer part of the number a.

Neural Networks

In this work, using neural networks in statistical identification in signal processing not only saves time, but also increases accuracy in solving the problem of identification. In this study, we used Dance and LSTM neural network, which is one of the popular recurrent networks.

EXPERIMENTS

For the experiment, we modeled the following signals and added white noise to it.

$$S_t = A\sin(2\pi t) \tag{5}$$

$$S_t = A\cos(2\pi t) \tag{6}$$

$$S_5 = A * (1/2 - |t - 1/2|) * (\cos(2p_1\pi t)\ \sin(2p_1\pi t)) \tag{7}$$

$$S_6 = \begin{cases} a_i & \text{if } i\text{th index is chosen randomly} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

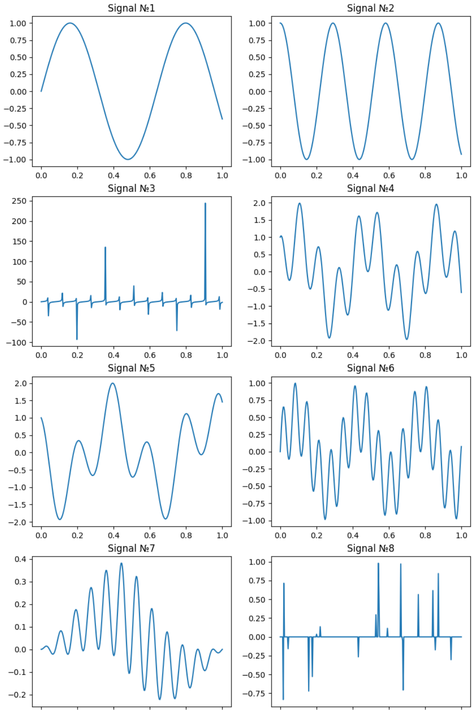
$$S_t = y_0 + (1 - x_{t-1})^p \sin(k_1\pi x_{t-1}) + (1 - x_{t-1})\cos(k_2\pi x_{t-1}) \tag{9}$$

$$S_t = y_0 + (1 - x_{t-1})^p \sin(k_1\pi x_{t-1}) - (1 - x_{t-1})\cos(k_2\pi x_{t-1}) \tag{10}$$

$$S_t = y_0 + (1 - x_{t-1})^p \sin(k_1\pi x_{t-1}) * (1 - x_{t-1})\cos(k_2\pi x_{t-1}) \tag{11}$$

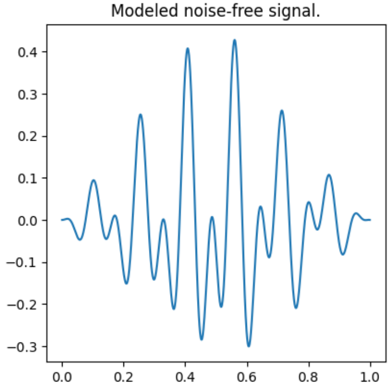
$$S_t = y_0 + (1 - x_{t-1})^p \sin(k_1\pi x_{t-1}) + x_{t-1}^p(1 - x_{t-1})\cos(k_2\pi x_{t-1})$$

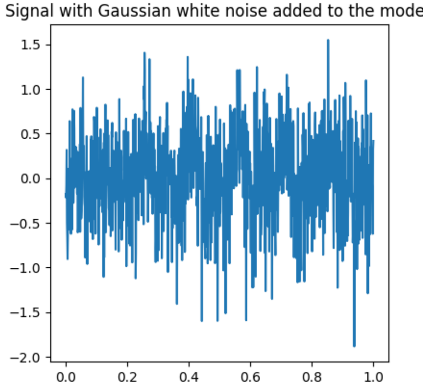
x=0..1,
$x_i = \frac{1}{n}$(12)


Signal №1
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00
0.0 0.2 0.4 0.6 0.8 1.0
Signal №2
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00
0.0 0.2 0.4 0.6 0.8 1.0
Signal №3
250 200 150 100 50 0 −50 −100
0.0 0.2 0.4 0.6 0.8 1.0
Signal №4
2.0 1.5 1.0 0.5 0.0 −0.5 −1.0 −1.5 −2.0
0.0 0.2 0.4 0.6 0.8 1.0
Signal №5
2.0 1.5 1.0 0.5 0.0 −0.5 −1.0 −1.5 −2.0
0.0 0.2 0.4 0.6 0.8 1.0
Signal №6
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00
0.0 0.2 0.4 0.6 0.8 1.0
Signal №7
0.4 0.3 0.2 0.1 0.0 −0.1 −0.2
Signal №8
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75

EXPERIMENTS

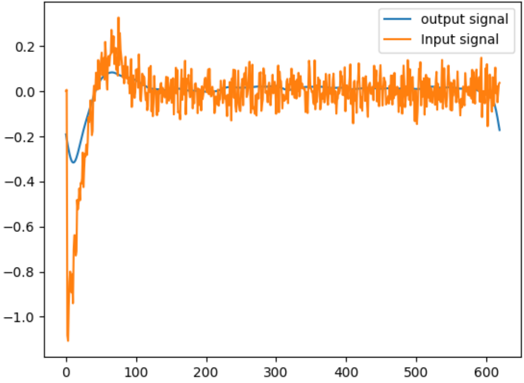
These signals are the input source for our neural network. You can see one of the generated signals below.





EXPERIMENTS

Such signals are evaluated using traditional methods. The evaluated signals are the result of the neural network. We present the initial incoming and outgoing data in a visual form as follows.



Experiments

Таблица: time

t \ ε	0.1	0.5	0.7
0.025	5.2037	3.4088	6.0244
0.01	34.7104	16.54	42.5601
0.005	137.2372	142.1254	139.0212
0.001	3386.7457	3302.5625	4094.2556

Experiments

Activation functions:
Linear activation function.
Sigmoid Function.
Tanh (Hyperbolic Tangent) Function.
Softmax Function

Neural Network model

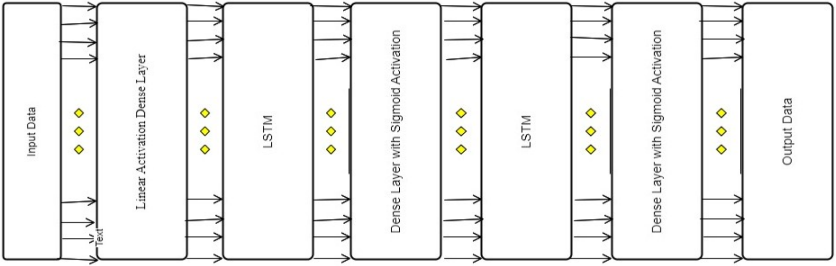
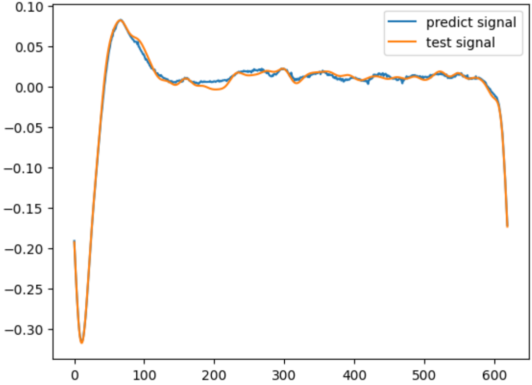


Рис.: Neural network model

Result

You can see the results of the prediction made using the artificial neural network in the figure below.



Results:

1. When artificial neural networks are used for signal processing, the time is reduced by more than 20 times.
2. As the database expands, so does the prediction accuracy.
3. The shortcomings of traditional methods can be filled by artificial neural networks.

Thank you for your attention!